SAFE HANDS & IIT-ian's PACE

MONTHLY MAJOR TEST-08 (JEE) ANS KEY Dt. 30-06-2023

PHYSICS	
Q. NO.	[ANS]
1	A
2	B
3	D
4	B
5	A
6	C
7	C
8	B
9	A
10	A
11	B
12	B
13	A
14	A
15	C
16	A
17	D
18	A
19	D
20	A
21	75.59
22	3
23	0
24	2
25	2
26	0.5
27	2
28	5
29	4.86
30	0.25

CHEMISTRY	
Q. NO.	[ANS]
31	A
32	C
33	B
34	A
35	C
36	B
37	D
38	A
39	B
40	A
41	B
42	D
43	A
44	A
45	B
46	A
47	D
48	D
49	A
50	A
51	2
52	4
53	800
54	5
55	3.74
56	1200
57	8
58	6
59	1
60	3

MATHS	
Q. NO.	[ANS]
61	C
62	B
63	C
64	B
65	A
66	D
67	A
68	B
69	A
70	C
71	A
72	C
73	D
74	C
75	B
76	D
77	C
78	D
79	B
80	C
81	4
82	21
83	9
84	2.5
85	4
86	252
87	5
88	1
89	0
90	7

SAFE HANDS & IIT-ian's PACE

MMT-08 (JEE)

Physics Answer key & Solutions

: ANSWER KEY :

1)	a	2)	b	3)	d	4)	b	21)	75.59	22)	3	23)	0	24)	2
5)	a	6)	c	7)	c	8)	b	25)	2	26)	0.5	27)	2	28)	5
9)	a	10)	a	11)	b	12)	b	29)	4.86	30)	0.25				
13)	a	14)	a	15)	c	16)	a								
17)	d	18)	a	19)	d	20)	a								

: HINTS AND SOLUTIONS :

Single Correct Answer Type

2 **(b)**

In the given system,

$$a=\frac{m_1-m_2}{m_1+m_2}=\frac{g}{8}$$

$$\therefore \frac{m_1-m_2}{m_1+m_2}=\frac{1}{8}$$

$$8m_1-8m_2=m_1+m_2$$

$$7m_1=9m_2$$

$$\frac{m_1}{m_2}=\frac{9}{7}$$

3 **(d)**

Using law of conservation of energy

$$-\frac{GMm}{r}=\frac{1}{2}mv^2-\frac{GMm}{R}$$

$$\frac{v^2}{2}=\frac{GM}{R}-\frac{GM}{r}$$

$$=GM\left(\frac{r-R}{rR}\right)=gR\left(\frac{r-R}{r}\right)$$

$$v=\sqrt{2gR(r-R)/r}$$

5 **(a)**

$$V=\frac{\sum q}{4\pi\varepsilon_0 r}=\frac{-10+10}{4\pi\varepsilon_0 r}=0$$

6 **(c)**

The capacitance of air capacitor

$$C_0=\frac{A\varepsilon_0}{d}=3\mu\text{F} \quad ...(i)$$

When a dielectric of permittivity ε_r and dielectric constant K is introduced between the plates of the capacitor, then its capacitance

$$C=\frac{KA\varepsilon_0}{d}=15\ \mu\text{F} \quad ...(ii)$$

Dividing Eq. (ii) by Eq. (i)

$$\frac{C}{C_0}=\frac{\frac{KA\varepsilon_0}{d}}{\frac{A\varepsilon_0}{d}}=\frac{15}{3}$$

$$\therefore \quad K=5$$

Permittivity of the medium

$$\varepsilon_r=\varepsilon_0 K$$

$$=8.854\times10^{-12}\times5$$

$$=44.27\times10^{-12}$$

$$=0.44\times10^{-10}\text{C}^2\text{N}^{-1}\text{m}^{-2}$$

7 **(c)**

After combining, the volume remains same *i.e.*,

Volume of bigger drop= N × volume of smaller drop

Or

$$\frac{4}{3}\pi R^3=N\times\frac{4}{3}\pi r^3$$

Or

$$N=\left(\frac{R}{r}\right)^3 \ldots\ldots\ldots\ldots\ldots (i)$$

As charge is conserved, hence

$$Q=Nq \ldots\ldots\ldots\ldots\ldots (ii)$$

Capacity of bigger drop $=4\pi\varepsilon_0 R$

Capacity of smaller drop$=4\pi\varepsilon_0 r$

From Eq. (ii), we have

$$(4\pi\varepsilon_0 R)V_{big}=N(4\pi\varepsilon_0 r)V_{small}$$

or $(4\pi\varepsilon_0 R)\times40=N(4\pi\varepsilon_0 r)\times10$

or $4R=Nr$

or $\frac{R}{r}=\frac{N}{4} \quad \ldots\ldots\ldots\ldots (iii)$

From Eqs. (i) and (iii), we have

$$N=\left(\frac{N}{4}\right)^3$$

or

$$N=\frac{N^3}{64}$$

or

$$N^2=64$$

OR N=8

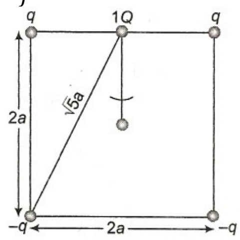
8 **(b)**

Both points are at same distance from the charge

9 **(a)**

$$U_i=\frac{2kqQ}{a}+\frac{2k(-q)Q}{\sqrt{5}a}=\frac{1}{4\pi\varepsilon_0}.\frac{2qQ}{a}(1-\frac{1}{\sqrt{5}})$$

$$U_f=0$$



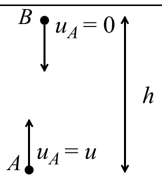
By conservation of energy

Gain in KE=loss in PE

$$K=\frac{1}{4\pi\varepsilon_0}.\frac{2qQ}{a}(1-\frac{1}{\sqrt{5}})$$

10 **(a)**

At time t



Velocity of $A, v_A = u - gt$ upward
Velocity of $B, v_B = gt$ downward
It we assume that height h is smaller than or equal to the maximum height reached by A, then at every instant v_A and v_B are in opposite directions
$\therefore V_{AB} = v_A + V_B$
$= u - gt + gt$ [Speeds in opposite directions get added]
$= u$

11 **(b)**
Pressure inside the mines is greater than that of normal pressure. Also we know that boiling point increases with increase in pressure

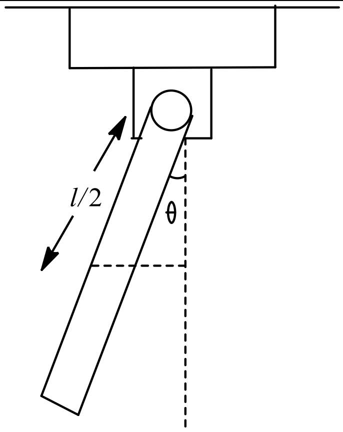
12 **(b)**
Due to centrifugal force

14 **(a)**
There are four beats between P and Q, therefore the possible frequencies of P or 254 (that is 250 ± 4)Hz. When the prong of P is field, its frequency become greater than the original frequency.
If we assume that the original frequency of P is 254, then on filing its frequency will be greater than 254. The beats between P and Q will be more than 4. But it is given that the beats are reduced to 2, therefore, 254 is not possible. Therefore, the required frequency must be 246 Hz.

15 **(c)**
Quantity C has maximum power. So it brings maximum error in P

16 **(a)**
$I_{\text{remaining}} = I_{\text{whole}} - I_{\text{removed}}$
or $I = \frac{1}{2}(9M)(R^2) - \left[\frac{1}{2}m\left(\frac{R}{3}\right)^2 + \frac{1}{2}m\left(\frac{2R}{3}\right)^2\right]$ ...(i)
Here, $m = \frac{9M}{\pi R^2} \times \pi\left(\frac{R}{3}\right)^2 = M$
Substituting in Eq. (i), we have
$I = 4MR^2$

17 **(d)**
Time period of simple pendulum is given by
$$T_1 = 2\pi\sqrt{\frac{l}{g}}$$
and time period of uniform rod in given position is given by
$$T_2 = 2\pi\sqrt{\frac{\text{Inertia factor}}{\text{spring factor}}}$$



Here, inertia factor=moment of inertia of rod at one end
$$= \frac{ml^2}{12} + \frac{ml^2}{4} = \frac{ml^2}{3}$$
Spring factor=restoring torque per unit angular displacement
$$= mg \times \frac{l}{2}\frac{\sin\theta}{\theta}$$
$$= mg \times \frac{l}{2}$$ (if θ is small)
$$\therefore \quad T_2 = 2\pi\sqrt{\frac{ml^2/3}{mgl/2}} = 2\pi\sqrt{\frac{2}{3}\frac{l}{g}}$$
Hence, $\frac{T_1}{T_2} = \sqrt{\frac{3}{2}}$

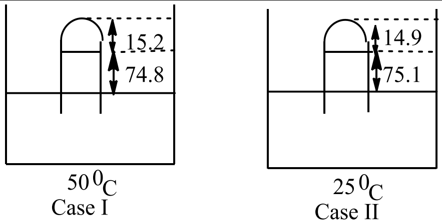
18 **(a)**
Mayer Formula

19 **(d)**
Excess pressure inside a spherical drop of water
$p = \frac{2T}{R}$
Given, $p_1 = 4p_2$
$\frac{2T}{R_1} = 4 \times \frac{2T}{R_2}$
or $R_2 = 4R_1$
Now, $\frac{m_1}{m_2} = \frac{4\pi R_1^3 d_1}{4\pi R_2^3 d_2}$
or $\frac{m_1}{m_2} = \frac{R_1^3}{R_2^3}$
$\frac{m_1}{m_2} = \frac{1}{64}$

20 **(a)**
Let initial kinetic energy, $E_1 = E$
Final kinetic energy, $E_2 = E + 300\%$ of $E = 4E$
As $P \propto \sqrt{E} \Rightarrow \frac{P_2}{P_1} = \sqrt{\frac{E_2}{E_1}} = \sqrt{\frac{4E}{E}} = 2 \Rightarrow P_2 = 2P_1$
$\Rightarrow P_2 = P_1 + 100\%$ of P_1
i.e., Momentum will increase by 100%

Integer Answer Type

21 **(75.59)**



$P_1 = 75 - 74.8 = 0.2$ cm of Hg

$P_2 = (P' - 75.1)$cm of Hg

The length of barometer tube above mercury level is equivalent to volume of air in the tube in both cases.

$\therefore V_1 = 90 - 74.8 = 15.2$ and $V_2 = 14.9$

From equation of state, $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

(
$\because$ number of moles of gas is constant in the barome

$$\therefore \frac{0.2 \times 15.2}{(50+273)} = \frac{P_2 \times 14.9}{(25+273)}$$

$$\therefore P_2 = \frac{298 \times 0.2 \times 15.2}{323 \times 14.9} = \frac{298 \times 152 \times 0.2}{323 \times 149}$$

$$= \frac{2 \times 8 \times 0.2}{17}$$

$$= \frac{3.2}{17}$$

$\therefore P_2 = 0.188 = 0.19$

$\therefore P' = 75.4 + 0.19 \approx 75.59$ cm of Hg

22 **(3)**
Electrostatic force will balance the surface tension force,

$$\frac{q_2}{a^2} \propto \gamma a \quad \Rightarrow a^3 \propto \frac{q^2}{\gamma}$$

$$\Rightarrow \quad a \propto \left(\frac{q^2}{\gamma}\right)^{1/3} \Rightarrow a = k\left(\frac{q^2}{\gamma}\right)^{1/3} \Rightarrow N = 3$$

23 **(0)**
When shells are connected, charge on inner shell will be transferred to the outer shell.

$$V_i = \frac{KQ}{2R} + \frac{K(2Q)}{2R} = \frac{3KQ}{2R}$$

$$V_f = \frac{K(3Q)}{2R} \to \Delta V = V_f - V_i = 0$$

24 **(2)**

$$a_1 = \frac{qE}{m} \text{and } a_2 = \frac{qE}{2m} \Rightarrow a_1 = 2a_2$$

$$K_1 = \frac{1}{2}m_1v_1^2 = \frac{1}{2}m_1(a,t)^2, K_2 = \frac{1}{2}m_2v_2^2$$

$$= \frac{1}{2}m_2(a_2t)^2$$

$$\frac{K_1}{K_2} = \frac{m_1a_1^2}{m_2a_2^2} = \frac{m}{2m} \times \frac{4}{1} = \frac{2}{1}$$

25 **(2)**

$$\rho = kr^a$$

$$E\left(r = \frac{R}{2}\right) = \frac{1}{8}E(r = R)$$

$$\frac{q_{\text{enclosed}}}{4\pi\varepsilon_0(R/2)^2} = \frac{1}{8}\frac{Q}{4\pi\varepsilon_0 R^2}$$

$$32q_{\text{enclosed}} = Q$$

$$q_{\text{enclosed}} = \int_0^{R/2} kr^a 4\pi r^2 dr = \frac{4\pi k}{(a+3)}\left(\frac{R}{2}\right)^{(a+3)}$$

$$Q = \frac{4\pi k}{(a+3)}R^{(a+3)}$$

$$\frac{Q}{q_{\text{enclosed}}} = 2^{a+3} \Rightarrow 2^{a+3} = 32 \Rightarrow a = 2$$

26 **(0.5)**
Let r be radius of wire, l be its length, Δr be change in r and Δl be the change in l when the wire is subjected to tension.

$$V_1 = \pi r^2 l$$

As length of the wire increases, its radius decreases,

$\therefore$ Volume of wire after elongation is,

$$V_2 = \pi r(r - \Delta r)^2(l + \Delta l)$$

Given : $V_1 = V_2$

$$\therefore \pi r^2 l = \pi r(r - \Delta r)^2(l + \Delta l)$$

$$= \pi[r^2 - 2r(\Delta r) + (\Delta r)^2](l + \Delta l)$$

$$= \pi r^2(l + \Delta l) - 2\pi r\Delta r(l + \Delta l) + \pi(\Delta r)^2(l + \Delta l)$$

Since, Δr *and* Δl are very small, terms of order $(\Delta r \times \Delta l)$ *and* $(\Delta r)^2$ and higher can be ignored.

$$\therefore \pi r^2 l = \pi r^2 l + \pi r^2 \Delta l - 2\pi r l \Delta r$$

$$\therefore r\Delta l = 2l\Delta r \Rightarrow \frac{\Delta l}{l} = 2\frac{\Delta r}{r}$$

$$\therefore \sigma = \frac{\Delta r/r}{\Delta l/l} = \frac{1}{2} = 0.5$$

Alternate method

We know that,

$$\frac{dV}{V}=(1-2\sigma)\frac{dL}{L}$$

$\therefore$ When $\frac{dV}{V}=0, then\ \sigma=+\frac{1}{2}=0.5$

27 **(2)**

$l=15.0$ m, $v=12$ ms^{-1}

Since there are 6 nodes, with the ends as nodes there will be five half wavelength in the string

So, $\frac{5\lambda}{2}=l=15\Rightarrow\lambda=6.0$ m

Using $f=\frac{v}{\lambda}=\frac{12}{6}=2.0$ Hz

28 **(5)**

When the string is cut, the weight of the rod constitutes torque about the hinge, so

$\tau_A=mg\frac{l}{2}$ (i)

According to Newton's second law,

$\tau_A=I\alpha$ (ii)

Where α is the angular acceleration of the rod about the end A. From Eqs. (i) and (ii), we get

$$I\alpha=mg\frac{l}{2}$$

Or $\alpha=\frac{mg\frac{l}{2}}{I}$

Here $I=ml^2/3$, therefore

$$\therefore\alpha=\frac{mgl/2}{ml^2/3}=\frac{3}{2}\frac{g}{l}$$

Acceleration of the CM of the rod is

$$a_{CM}=\alpha r=\frac{3}{2}\frac{g}{l}\times\frac{l}{2}=\frac{3g}{4}$$

Again by Newton's second law,

$mg-R_A=ma_{CM}$

Or $mg-R_A=m\times\frac{3g}{4}$

$$\therefore R_A=\frac{Mg}{4}=\frac{2\times10}{4}=5\text{ N}$$

29 **(4.86)**

The speed at the highest point must be

$v\geq\sqrt{rg}$

Now $v=r\omega=r\,(2\pi/T)$

$\therefore\ r\,(2\pi/T)\geq\sqrt{rg}$

$\therefore\ T\leq\frac{2\pi r}{\sqrt{rg}}=2\pi\sqrt{\frac{r}{g}}$

$\therefore\ T_{max}=2\pi\sqrt{\frac{6}{10}}$

$\therefore\ T_{max}=4.86$ s

30 **(0.25)**

Terminal velocity of 1st ball,

$$(v_0)_1=\frac{2}{9}\times\frac{r^2(\rho_1-\sigma_{water})}{\eta_{water}}$$

Terminal velocity of 2ndball,

$$(v_0)_2=\frac{2}{9}\times\frac{r^2(\rho_2-\sigma_{liq})}{\eta_{liq}}$$

For $(v_0)_1=(v_0)_2$

$$\frac{\eta_{water}}{\eta_{liq}}=\frac{(\rho_1-\sigma_{water})}{(\rho_2-\sigma_{liq})}=\frac{2.7-1}{8.4-1.6}=\frac{1.7}{6.8}=0.25$$

SAFE HANDS & IIT-ian's PACE

MMT-08 (JEE) Maths Answer key & Solutions

: ANSWER KEY :

61)	c	62)	b					84)	2.5
63)	c	64)	b	65)	a	66)	d	85)	4
67)	a	68)	b	69)	a	70)	c	86)	252
71)	a	72)	c	73)	d	74)	c	87)	5
75)	b	76)	d	77)	c	78)	d	88)	1
79)	b	80)	c					89)	0
81)	4							90)	7
82)	21								
83)	9								

: HINTS AND SOLUTIONS :

Single Correct Answer Type

61 **(c)**

Since, $\sin^2\frac{A}{2}, \sin^2\frac{B}{2}, \sin^2\frac{C}{2}$ be in HP

$\Rightarrow \frac{1}{\sin^2\frac{A}{2}}, \frac{1}{\sin^2\frac{B}{2}}, \frac{1}{\sin^2\frac{C}{2}}$ are in AP

$\Rightarrow \frac{1}{\sin^2\frac{C}{2}} - \frac{1}{\sin^2\frac{B}{2}} = \frac{1}{\sin^2\frac{B}{2}} - \frac{1}{\sin^2\frac{A}{2}}$

$\Rightarrow \frac{ab}{(s-a)(s-b)} - \frac{ac}{(s-a)(s-c)}$

$= \frac{ac}{(s-a)(s-c)} - \frac{bc}{(s-b)(s-c)}$

$\Rightarrow \left(\frac{a}{s-a}\right)\left(\frac{b(s-c)-c(s-b)}{(s-b)(s-c)}\right)$

$= \left(\frac{c}{s-c}\right)\left(\frac{a(s-b)-b(s-a)}{(s-a)(s-b)}\right)$

$\Rightarrow abs - abc - acs + abc = acs - abc - bcs + abc$

$\Rightarrow ab - ac = ac - bc \Rightarrow ab + bc = 2ac$

$\Rightarrow \frac{1}{c} + \frac{1}{a} = \frac{2}{b}$

$\Rightarrow a, b, c$ are in HP

62 **(b)**

Since, $\left|\frac{z}{z-i/3}\right|=1$

$\Rightarrow 3|z| = |3z - i|$

$\Rightarrow 3|x+iy| = |3(x+iy) - i|$ [put $z = x + iy$]

$\Rightarrow 3\sqrt{x^2+y^2} = \sqrt{(3x)^2 + (3y-1)^2}$

$\Rightarrow 9x^2 + 9y^2 = 9x^2 + 9y^2 + 1 - 6y$

$\Rightarrow y = \frac{1}{6}$

Which shows that z lies on a straight line.

63 **(c)**

$(1+x)^{21} + (1+x)^{22} + \ldots + (1+x)^{30}$

$= (1+x)^{21}[1 + (1+x)^1 + \ldots + (1+x)^9]$

$= (1+x)^{21}\left[\frac{(1+x)^{10}-1}{(1+x)-1}\right]$

$= \frac{1}{x}[(1+x)^{31} - (1+x)^{21}]$

$\therefore$ Coefficient of x^5 in the given expression

= Coefficient of x^5 in $\frac{1}{x}[(1+x)^{31} - (1+x)^{21}]$

= Coefficient of x^6 in $[(1+x)^{31} - (1+x)^{21}]$

$= {}^{31}C_6 - {}^{21}C_6$

65 **(a)**

Given, $4P(A) = 6P(B) = 10P(A\cap B) = 1$

$\therefore P\left(\frac{B}{A}\right) = \frac{P(A\cap B)}{P(A)} = \frac{\frac{1}{10}}{\frac{1}{4}} = \frac{2}{5}$

66 **(d)**

We have,

$\lim_{x\to 0} \frac{\log_e(3+x) - \log_e(3-x)}{x} = k$

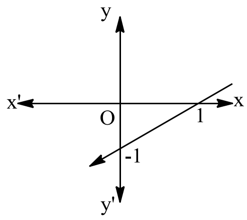
$\Rightarrow \lim_{x\to 0} \frac{\log_e\left(1+\frac{x}{3}\right) - \log_e\left(1-\frac{x}{3}\right)}{x} = k$

$\Rightarrow \lim_{x\to 0} \frac{1}{3} \times \frac{\log_e\left(1+\frac{x}{3}\right)}{\frac{x}{3}} + \frac{1}{3} + \lim_{x\to 0} \frac{\log\left(1-\frac{x}{3}\right)}{\left(-\frac{x}{3}\right)} = k$

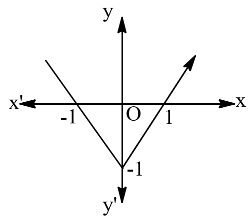
$\Rightarrow \frac{1}{3} + \frac{1}{3} = k \Rightarrow k = \frac{2}{3}$

67 **(a)**

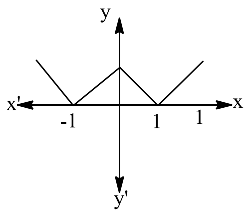
Using graphical transformation



(i) y = x - 1



(ii) y = |x| - 1



(iii) y = ||x| - 1|

As, we know the function is not differentiable at6 sharp edges and in figure (iii) $y = \big||x| - 1\big|$ we have 3 sharp edges at $x = -1, 0, 1$

$\therefore f(x)$ is not differentiable at $\{0, \pm 1\}$

68 **(b)**

The required equation of circle is

$(x^2 + y^2 - 6) + \lambda(x^2 + y^2 - 6y + 8) = 0$ ...(i)

It passes through (1, 1)

$\therefore (1 + 1 - 6) + \lambda(1 + 1 - 6 + 8) = 0$

$\Rightarrow -4 + 4\lambda = 0$

$\Rightarrow \lambda = 1$

$\therefore$ required equation of circle is

$x^2 + y^2 - 6 + x^2 + y^2 - 6y + 8 = 0$

$\Rightarrow 2x^2 + 2y^2 - 6y + 2 = 0$

$\Rightarrow x^2 + y^2 - 3y + 1$

69 **(a)**

Length of the chord

$= \sqrt{[4\cos(\theta+60°) - 4\cos\theta]^2 + [4\sin(\theta+60°) - 4\sin\theta]^2}$

$= 4\sqrt{\cos^2(\theta+60°) + \cos^2\theta + \sin^2(\theta+60°) + \sin^2\theta - 2\cos(\theta+60°)\cos\theta - 2\sin(\theta+60°)\sin\theta}$

$= 4\sqrt{1+1-2\cos 60°} = 4$

71 **(a)**

Given , $\frac{x^2}{16} + \frac{y^2}{9} = 1$

$\therefore e = \sqrt{1 - \frac{9}{16}} = \frac{\sqrt{7}}{4}$

$\therefore$ Coodinates of foci are $(\pm\sqrt{7}, 0)$

Since, centre of circle is (0, 3) and passing through foci $(\pm 7, 0)$

$\therefore$ Radius of circle= $\sqrt{(0 \pm \sqrt{7})^2 + (3-0)^2}$

$= \sqrt{7+9} = 4$

72 **(c)**

We have,

$75600 = 2^4 \cdot 3^3 \cdot 5^2 \cdot 7$

The total number of ways of selecting some or all out of four 2's, three 3's, two 5's and one 7's

$= (4+1)(3+1)(2+1)(1+1) - 1 = 119$

But, this includes the given number itself. Therefore, the required number of proper factors is 118

73 **(d)**

We have,

$x^{(\log_{10} x)^2 - 3(\log_{10} x) + 1} > 10^3$

$\Rightarrow (\log_{10} x)^2 - 3(\log_{10} x) + 1 > \log_x 10^3$

$\Rightarrow (\log_{10} x)^2 - 3(\log_{10} x) + 1 > \frac{3}{\log_{10} x}$

$\Rightarrow \frac{(\log_{10} x)^3 - 3(\log_{10} x)^2 + (\log_{10} x) - 3}{\log_{10} x} > 0$

$\Rightarrow \frac{\{(\log_{10} x)^2 + 1\}(\log_{10} x - 3)}{\log_{10} x} > 0$

$\Rightarrow \frac{(\log_{10} x - 3)}{(\log_{10} x - 0)} > 0$

$\Rightarrow \log_{10} x < 0$ or, $\log_{10} x > 3$

$\Rightarrow x < 1$ or, $x > 10^3$

$\Rightarrow x \in (0,1) \cup (10^3, \infty)$ [$\because \log_{10} x$ is defined for $x > 0$]

74 **(c)**

The equations $ax + by + c = 0$ and $dx + ey + f = 0$ will represent the same straight line if their slopes and y-intercepts are equal

$\therefore -\frac{a}{b} = -\frac{d}{e}$ and $-\frac{c}{b} = -\frac{f}{e}$

$\Rightarrow \frac{a}{d} = \frac{b}{e}$ and $\frac{b}{e} = \frac{c}{f} \Rightarrow \frac{a}{d} = \frac{b}{e} = \frac{c}{f}$

75 **(b)**

$y = a\cos(\log x) + b\sin(\log x)$

On differentiating w.r.t. x, we get

$y' = \frac{-a\sin(\log x)}{x} + \frac{b\cos(\log x)}{x}$

$\Rightarrow xy' = -a\sin(\log x) + b\cos(\log x)$

Again, on differentiating w.r.t. x, we get

$xy'' + y' = -a\cos(\log x)\frac{1}{x} - b\sin(\log x)\frac{1}{x}$

$\Rightarrow x^2y'' + y'x = -[a\cos(\log x) + b\sin(\log x)]$

$\Rightarrow x^2y'' + xy' = -y$

76 **(d)**

Let $\cos^{-1} x = \theta$. Then, $x = \cos\ \theta$

Also,

$-1 \le x \le 0 \Rightarrow -1 \le \cos\theta \le 0 \Rightarrow \frac{\pi}{2} \le \theta \le \pi$

Now,

$\cos^{-1}(2x^2 - 1) = \cos^{-1}(\cos 2\,\theta)$

$= \cos^{-1}(2\,\pi - 2\,\theta)$

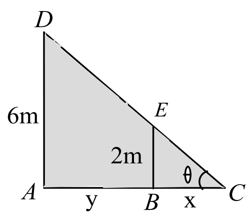
$\Rightarrow \cos^{-1}(2x^2 - 1)$

$= 2\,\pi - 2\,\theta \left[\because \frac{\pi}{2} \le \theta \le \pi \Rightarrow \pi \le 2\theta \le 2\pi \Rightarrow 0 \le 2\pi - 2\theta \le \pi\right]$

$\Rightarrow \cos^{-1}(2x^2 - 1) = 2\,\pi - 2\cos^{-1} x$

77 **(c)**

In ΔADC , $\tan\theta = \frac{6}{x+y}$



And in ΔBCE, $\tan\theta = \frac{2}{x}$

$\therefore \frac{2}{x} = \frac{6}{x+y} \Rightarrow y = 2x$

On differentiating w. r. t. t,we get

$\frac{dy}{dt} = 2\frac{dx}{dt} \Rightarrow \frac{dx}{dt} = 3km/h \quad \left[\because \frac{dy}{dt} = 6, \text{given}\right]$

78 **(d)**

$We\ have\ , y = x + \frac{4}{x^2}$

On differencing with respect to x, we get

$\frac{dy}{dx} = 1 - \frac{8}{x^3}$

Since, the tangent is parallel to $x - axis$, therefore

$\frac{dy}{dx} = 0$

$\Rightarrow \quad x^3 = 8 \quad \Rightarrow x = 2\ and\ y = 3$

79 **(b)**

Given, $2\sin^2\frac{\theta}{2} = 2\sin\frac{\theta}{2}.\cos\frac{\theta}{2}.\sin\frac{\theta}{2}$

$\Rightarrow 2\sin^2\frac{\theta}{2}\left[1 - \cos\frac{\theta}{2}\right] = 0$

$\Rightarrow \sin\frac{\theta}{2} = 0$ or $2\sin^2\frac{\theta}{4} = 0$

$\Rightarrow \frac{\theta}{2} = k\pi$ or $\frac{\theta}{4} = k\pi$

Hence, $\theta = 2k\pi$ or $\theta = 4k\pi,\ k \in I$

80 **(c)**

The function $f(x) = {}^{7-x}P_{x-3}$ is defined only if x is an integer satisfying the following inequalities:

(i)$7 - x \geq 0$ (ii)$x - 3 \geq 0$ (iii)$7 - x \geq x - 3$

Now,

$$\left.\begin{array}{r} 7 - x \geq 0 \Rightarrow x \leq 7 \\ x - 3 \geq 0 \Rightarrow x \geq 3 \\ 7 - x \geq x - 3 \Rightarrow x \leq 5 \end{array}\right\} \Rightarrow 3 \leq x \leq 5$$

Hence, the required domain is $\{3, 4, 5\}$

Integer Answer Type

81 **(4)**

$x^{1/8} = (3x^4 + 4)^{1/64} \Rightarrow x^8 = 3x^4 + 4 \Rightarrow x^4 = 4$

52 **(21)**

${}^{n}C_{r-2} : {}^{n}C_{r-1} : {}^{n}C_{r} = 1:3:5$

$\Rightarrow \frac{{}^{n}C_{r-2}}{{}^{n}C_{r-1}} = \frac{1}{3}$ and $\frac{{}^{n}C_{r-1}}{{}^{n}C_{r}} = \frac{3}{5}$

$\Rightarrow \frac{r-1}{n-r+2} = \frac{1}{3}$ and $\frac{r}{n-r+1} = \frac{3}{5}$

$\Rightarrow 4r - n = 5$ ....(i)

and $8r - 3n = 3$......(ii)

Solving (i) and (ii), we get $n = 7$ and $r = 3$

$\Rightarrow$ Average of coefficients $= \frac{{}^7C_1 + {}^7C_2 + {}^7C_3}{3}$

$= \frac{7 + 21 + 35}{3}$

$= 21$

83 **(9)**

$$\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$$

$R_1 \rightarrow R_1 + R_2 + R_3$

$$\begin{vmatrix} x+9 & x+9 & x+9 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$$

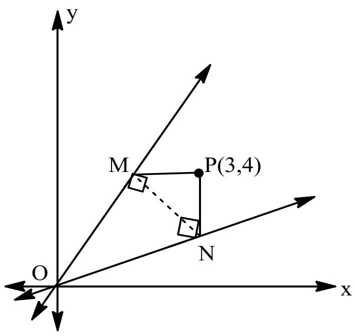
$$\Rightarrow (x+9)\begin{vmatrix} 1 & 1 & 1 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$$

$\Leftrightarrow (x+9)(x^2 - 12 + 14 - 2x + 12 - 7x) = 0$

$\Leftrightarrow (x+9)(x^2 - 9x + 14) = 0$

$\Rightarrow$ Sum of the other roots $= 9$

84 **(2.5)**



□ONPM is a cyclic quadrilateral, inscribed in a circle having OP as a diameter.

$\Rightarrow$ Radius of $\Delta PMN = \frac{1}{2}|OP|$

$= \frac{1}{2}\sqrt{4^2 + 3^2}$

$= \frac{5}{2} = 2.5$

85 **(4)**

$|z| = 2 \Rightarrow x^2 + y^2 = 4 \quad$ ... (i)

Let $w = z + \frac{1}{z}$

$$= x + iy + \frac{1}{x + iy}$$

$$= x + iy + \frac{x - iy}{x^2 + y^2}$$

$$= x + iy + \frac{x - iy}{4} \quad \text{... [From (i)]}$$

Let $w = u + iv$. Then

$$u = \frac{5x}{4}, v = \frac{3y}{4}$$

$$x^2 + y^2 = 4 \Rightarrow \left(\frac{4u}{5}\right)^2 + \left(\frac{4v}{3}\right)^2 = 4$$

$$\Leftrightarrow \frac{u^2}{\left(\frac{25}{4}\right)} + \frac{v^2}{\left(\frac{9}{4}\right)} = 1$$

$$\Rightarrow a^2 = \frac{25}{4}, b^2 = \frac{9}{4}$$

$$\Rightarrow a^2 - b^2 = \frac{25 - 9}{4} = 4$$

86 **(252)**

Let $x_1, x_2, x_3, \dots, x_6$ be the number of pencils received by 6 kids.

Then $x_1 + x_2 + \cdots + x_6 = 11$,
$x_i \geq 1, i = 1,2, \dots, 6$
Take $x_i = a_i + 1, i = 1,2, \dots, 6$
where each $a_i \geq 0$
Substituting in equation (i), we get
$6 + a_1 + a_2 + \cdots + a_6 = 11$
$\Leftrightarrow a_1 + a_2 + \cdots + a_6 = 5$,
$a_i \geq 0, i = 1,2, \dots, 6$
Number of integer solutions of system (i)
= Number of non-negative integer solutions of (ii)
$= {}^{6+5-1}C_5 = {}^{10}C_5$
$\Rightarrow$ Required number of ways
$= {}^{10}C_5 = 252$

87 **(5)**

We have $(g\circ f)(x) = x$
$\Rightarrow g'\big(f(x)\big)f'(x) = 1$
When $f(x) = -\frac{7}{6} \Rightarrow x = 1$
$\Rightarrow g'\big(f(x)\big)g'\left(-\frac{7}{6}\right)f'(1) = 1$
Hence $g'\left(-\frac{7}{6}\right) = \frac{1}{f'(1)} = \frac{1}{5}$

88 **(1)**

Given expression is defined only for $x = 1$ and -1

$\therefore f(1) = 1$ and $f(-1) = (1 + \pi)(1 + \pi) = (1 + \pi)^2$

Hence, the least value is 1

89 **(0)**

$$\begin{vmatrix} 3u^2 & 2u^3 & 1 \\ 3v^2 & 2v^3 & 1 \\ 3w^2 & 2w^3 & 1 \end{vmatrix} = 0$$

$R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$

$$\Rightarrow \begin{vmatrix} u^2 - v^2 & u^3 - v^3 & 0 \\ v^2 - w^2 & v^3 - w^3 & 0 \\ w^2 & w^3 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} u + v & u^2 + v^2 + vu & 0 \\ v + w & v^2 + w^2 + vw & 0 \\ w^2 & w^3 & 1 \end{vmatrix} = 0$$

$R_1 \to R_1 - R_2$

$$\Rightarrow \begin{vmatrix} u - w & (u^2 - w^2) + v(u - w) & 0 \\ v + w & v^2 + w^2 + vw & 0 \\ w^2 & w^3 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & u + w + v & 0 \\ v + w & v^2 + w^2 + vw & 0 \\ w^2 & w^3 & 1 \end{vmatrix} = 0$$

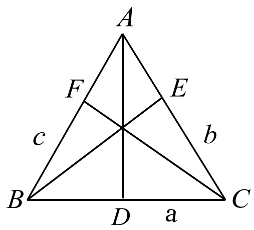
$\Rightarrow (v^2 + w^2 + vw) - (v + w)[(v + w) + u] = 0$
$\Rightarrow v^2 + w^2 + vw - (v + w)^2 - u(v + w) = 0$
$\Rightarrow uv + vw + wu = 0$

90 **(7)**

$$\Delta = \frac{1}{2} \times 210a = \frac{1}{2} \times 195b = \frac{1}{2} \times 182c$$

$$\therefore b = \frac{210a}{195} = \frac{14}{13}a; c = \frac{210a}{182} = \frac{15}{13}a$$

Hence, $2s = a + \frac{14a}{13} + \frac{15a}{13} = \frac{(13+14+15)a}{13}$



$$\Rightarrow s = \frac{21a}{13}$$

Also $\Delta = \sqrt{\frac{21a}{13}\left(\frac{8a}{13}\right)\left(\frac{7a}{13}\right)\left(\frac{6a}{13}\right)} = \frac{84a^2}{169}$

But $\Delta = \frac{1}{2} \times 210a = \frac{84a^2}{169}$

$$\Rightarrow a = \frac{105 \times 169}{84} = \frac{15 \times 169}{12} = \frac{845}{4}$$